

Traffic jams and ordering far from thermal equilibrium

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Abstract

The recently suggested correspondence between domain dynamics of traffic models and the asymmetric chipping model is reviewed. It is observed that in many cases traffic domains perform the two characteristic dynamical processes of the chipping model, namely chipping and diffusion. This correspondence indicates that jamming in traffic models in which all dynamical rates are non-deterministic takes place as a broad crossover phenomenon, rather than a sharp transition. Two traffic models are studied in detail and analyzed within this picture.

Key words: Stochastic Processes, Phase Separation, Transportation

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1 Introduction

Ordering in one-dimensional systems far from thermal equilibrium has been studied extensively in recent years [1–3]. In the case of thermal equilibrium, it is well known that one-dimensional systems with short-range interactions cannot exhibit long range order. In contrast, it has been repeatedly demonstrated that non-equilibrium driven systems, whose dynamics does not obey detailed-balance, can be ordered even when the dynamics is local and noisy.

Single-lane traffic constitutes a particularly interesting class of one-dimensional driven systems. Modeling traffic dynamics has been of considerable interest over the years [4,5]. A very useful quantity which has often been used to characterize traffic flow is the relation between the density of cars in the road and the traffic throughput. This relation, termed the *fundamental diagram*, was measured empirically in various situations, and was studied in a large variety of models [4–6]. At low car density one expects the traffic to flow smoothly

with a linear increase of the throughput with the car density. On the other hand at high densities jams are formed and the flow is lowered, sometimes even to a complete stop [6, 7]. One is interested in developing a better and deeper understanding of the dynamical mechanisms which are involved in this behavior. In particular, an intriguing question is whether jamming takes place via a genuine sharp phase transition at a particular car density, or perhaps it develops as a broad crossover phenomenon [4, 5, 8]. This question is closely related to the question of ordering in driven one-dimensional systems.

In recent years probabilistic Cellular Automata (CA) models have been introduced to analyze traffic flow [4, 5, 9, 10]. In such models both time and space are discrete and all dynamical variables (*e.g.* position and velocity of all cars) are updated simultaneously according to some update scheme. This provides a rather efficient way for carrying out numerical studies of the fundamental diagram. Some traffic CA models were suggested to exhibit jamming phase transitions [11–15]. However, the existence of such a transition can only be explicitly demonstrated in some limiting cases where certain dynamical processes are deterministic. These cases are less relevant for realistic traffic flow where all dynamical processes are expected to be noisy. The existence of a jamming phase transition in more generic cases where all dynamical processes are non-deterministic is a more difficult theoretical question.

Recently it has been suggested [16] that coarse-grained traffic dynamics can be modeled by the asymmetric Chipping Model (CM) [17–22]. This model, introduced a few years ago, yields a particular mechanism of condensation transition in one-dimensional models. The CM belongs to a class of urn models. These are simple lattice models, defined on a ring geometry, where each site can either be vacant or occupied by one or more particles. The dynamics of the CM involves two processes: *chipping*, where a single particle hops to a nearest neighbor site at a constant rate ω ; and *diffusion*, where all particles in a site hop together to an adjacent site with rate α . Mean-field analysis indicates that this model exhibits a condensation transition at a critical density. This result remains valid beyond the mean-field approximation as long as the chipping process is symmetric, with equal right and left hopping probabilities [19–21]. It has also been argued that if the chipping process is biased, no condensation takes place [22].

To make the correspondence between traffic models and the CM it has been suggested that the coarse-grained evolution of traffic models is described in terms of domain dynamics, which essentially involves diffusion and asymmetric chipping processes [16]. It has thus been concluded that no phase separation transition should be expected in this class of models, in the case where all transition rates are non-deterministic. In these cases jamming takes place as a broad crossover process rather than via a sharp phase transition.

In this paper we review the correspondence between traffic models and the asymmetric CM and closely examine the coarse-grained dynamics of some traffic models in view of this correspondence. In Section 2 we begin by introducing a simple traffic cellular automaton, whose dynamics can be mapped onto a zero range process (ZRP) [2,23]. In this urn model only chipping takes place but no diffusion. This particular traffic model is thus restricted in its dynamics. Nevertheless, the exact solution of this model yields an insight into the fundamental diagram which characterizes traffic models. A more general traffic model, which does correspond to the CM with both chipping and diffusion processes, is introduced and studied in Section 3. To examine this approach within a broader scope we apply it in Section 4 to a recently introduced traffic CA [24]. Our results are summarized in Section 5.

2 A simple traffic CA – chipping without diffusion

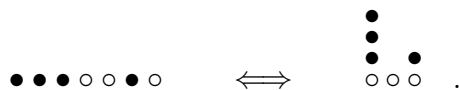
We start by considering an exactly soluble traffic model, whose domain dynamics involves only chipping processes, without diffusion. The steady-state properties of this model are obtained by a mapping onto a zero range process (ZRP) [2,23]. The model is defined on a periodic lattice of size L with $N = \rho L$ cars. It evolves in discrete time by simultaneously updating all sites using the transition probabilities

$$\bullet \bullet \circ \xrightarrow{r} \bullet \circ \bullet \qquad \circ \bullet \circ \xrightarrow{q} \circ \circ \bullet, \quad (1)$$

where \bullet denote a car and \circ a vacancy. This is a special case of the model considered in [24]. It is also closely related to the VDR model introduced and studied in [15]. With $r < q$ the model exhibits the 'slow-to-start' feature which characterizes many traffic models. This simple model belongs to a class of models where no explicit velocity variable is attached to each car (see *e.g.* [24–26]). Rather, cars progress according to rules which depend only on the position of neighboring cars. This class of models provides a useful tool for studying specific features of traffic models. Other more complex models introduce velocity variables, and define rules by which both velocity and position are updated [9, 10, 15, 16].

To proceed we define the ZRP dynamics, and argue that the dynamics (1) corresponds to a particular choice of its rates. The ZRP is an urn model, where particles hop between nearest neighbor urns with rates $\omega(k)$ which depends only on the number of particles k at the departure site. To map the traffic model onto a ZRP it is instructive to view each vacancy as an urn, occupied by the uninterrupted sequence of particles located at its left. For example, the following traffic configuration corresponds to 3 urns occupied by 3, 0, and 1

particles, respectively,



Therefore, the dynamics (1) corresponds to hopping of the topmost particle from an urn to its right neighbor, with rates

$$\omega(k) = \begin{cases} q & k = 1 \\ r & k > 1 \end{cases} . \quad (2)$$

The grand-canonical steady-state distribution of the ZRP in parallel dynamics is a product measure, whereby the single-site occupation weight is given by [2, 27]

$$f(k) = \begin{cases} 1 & k = 0 \\ \frac{z^k}{1-\omega(k)} \prod_{m=1}^k \frac{1-\omega(m)}{\omega(m)} & k > 0 \end{cases} . \quad (3)$$

Here z serves as a fugacity, which is set to determine the overall density. Using this solution, the un-normalized domain size (or jam length) distribution in the traffic model with $q \neq 1$ is

$$f(k) = \begin{cases} 1 & k = 0 \\ \frac{z}{q} & k = 1 \\ z^{\frac{1-q}{r}} \left(z \frac{1-r}{r} \right)^{k-1} & k \geq 2 \end{cases} . \quad (4)$$

The case $q = 1$ is unique, and will be treated below. The fundamental diagram is obtained by inserting (4) in the general expression for the current

$$J(z) = \frac{\sum_{k=1}^{\infty} \omega(k) f(k)}{\sum_{k=0}^{\infty} (k+1) f(k)} , \quad (5)$$

with the fugacity z determined from the equation $\phi = \sum_k k f(k) / \sum_k f(k)$. Here ϕ is the average jam-size, which is related to the car density ρ in the traffic model by $\phi = \rho / (1 - \rho)$.

It is readily seen from (4) that for $q \neq 1$ the jam-length distribution decays exponentially at any density, and thus no phase transition takes place. The fundamental diagram for various values of q at $r = 2/3$ is given in Fig. 1.

2.1 The deterministic case $q = 1$

At $q = 1$ isolated particles move deterministically. This is an example of the cruise control (CC) limit of traffic models, whereby cars moving at their

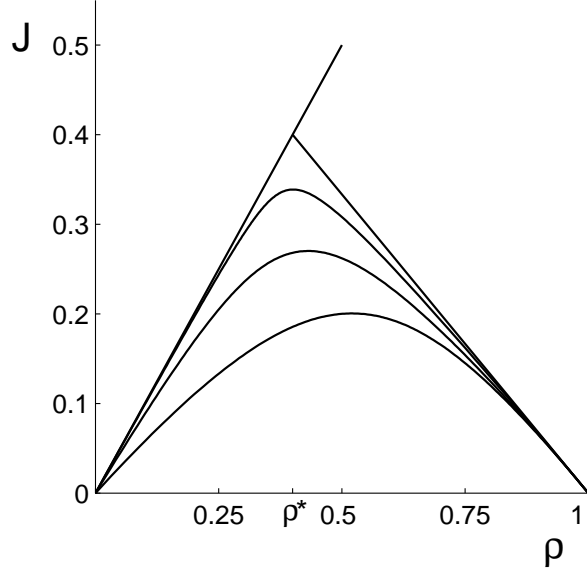


Fig. 1. The fundamental diagram of the simple model, for $r = 2/3$ and $q = 0.6, 0.9, 0.99$, and 1 from bottom to top.

maximal speed do not decelerate as long as they are not interrupted by other cars. We first address the low-density regime $\rho < 1/2$, where a free-flow state is reached for any finite system at sufficiently long time. In this state each car is separated from a neighboring car by at least one vacancy. This is an absorbing state, since once formed it remains unchanged. Using (4) one obtains the following domain-size distribution

$$f(k) = \begin{cases} 1 & k = 0 \\ z & k = 1 \\ 0 & \text{otherwise} \end{cases} . \quad (6)$$

The car density is then given by $\rho = z/(1 + 2z)$, implying $\rho \leq 1/2$, and a current $j(\rho) = \rho$. Clearly, at high densities this distribution does not describe the steady state, and one expects a jammed state, with a macroscopic jam coexisting with a free-flow region. Since this macroscopic jam emits particles with probability r , the particle density in the free flow region is just $r/(1 + r)$. It is straightforward to show that in this case the current is given by $j(\rho) = r(1 - \rho)$. The current-density relation in the two regimes is depicted in Fig. 1, where the two branches intersect at $\rho^* = r/(1 + r)$.

In the region $\rho^* < \rho < 1/2$ the two branches coexist. In this density interval any finite system will evolve into the free-flow branch in the long-time limit. In the following we show that in the thermodynamic limit the time it takes for the jammed state to reach the free-flow state grows exponentially with the system size. This implies that in fact in these densities *both* free-flow and jammed states are thermodynamically stable. To demonstrate this point, we consider a system of length L occupied by N cars, with initial condition whereby all

cars reside in a single domain. With $N/L < 1/2$ the system will eventually evolve into a free flow state. Let us estimate the time it takes to reach this state. Cars are emitted from the front end of the domain with probability r . Once a car is emitted, it moves deterministically with velocity 1, rejoining the domain at its back end after exactly $L - N$ time steps (assuming that the domain has not disintegrated in this time interval). The probability p_{dis} that a domain of size N disintegrates during a time interval of $L - N$ is

$$p_{\text{dis}} = \sum_{k=N-1}^{L-N} \binom{L-N}{k} r^k (1-r)^{L-N-k}. \quad (7)$$

For $\rho < \rho^*$ this probability is equal to 1 for large L (as the dominant term in the sum is obtained at some $k > N$), while in the interval $\rho^* < \rho < 1/2$ the sum is dominated by the $k = N - 1$ term. Hence,

$$p_{\text{dis}} \simeq \left[\frac{1-\rho}{1-2\rho} (1-r) \right]^{L-N} \left[\frac{\rho}{1-2\rho} \frac{1-r}{r} \right]^{-N}. \quad (8)$$

This probability is exponentially small in L , and therefore the decay time of the jammed state is exponentially long.

The fundamental diagram of this model, Fig. 1, has the characteristics of a more general class of traffic models. It exhibits a jamming transition at $\rho = \rho^*$ only in the cruise control limit $q = 1$ where at least one of the dynamical processes become deterministic. The free-flow and the jammed states coexist within some density interval as two thermodynamically stable states. Once all rates become noisy, the jamming transition disappears and the fundamental diagram becomes smooth.

3 Extended traffic CA – diffusion and chipping

We now consider a more general traffic CA, whose dynamics involves both chipping and diffusion processes. It is argued that the domain dynamics of this model is characteristic of many traffic models. The model is defined on a periodic lattice of size L , occupied by $N = \rho L$ cars. The transition probabilities are given by ¹

$$\bullet \circ \circ \xrightarrow{a} \circ \bullet \circ \quad \bullet \circ \circ \xrightarrow{u} \circ \circ \bullet \quad \bullet \circ \bullet \xrightarrow{s} \circ \bullet \bullet, \quad (9)$$

where as before, \bullet denotes a car and \circ a vacancy. In this model cars can move with either velocity 1 (the a and s processes) or with velocity 2 (the u process).

¹ A very similar model can be defined as a variant of the VDR model [15] in terms of velocities and deceleration probabilities. For simplicity we adopt the notation given above.

The fact that cars perform two types of move will be shown below to lead to both diffusion and chipping. As in other traffic models, the general features of the model are revealed only when the maximal velocity is larger than one.

In what follows we elaborate on the correspondence between this model and the asymmetric CM. Let us first define the CM more precisely, and summarize its main features. The CM is defined on a periodic lattice of size M , occupied by $N = \phi M$ particles. The dynamics is defined through the rates by which two nearest neighbor sites containing k and m particles, respectively, exchange particles:

$$(k, m) \xrightarrow{\alpha} (k + m, 0) \quad (10a)$$

$$(k, m) \xrightarrow{q\omega} (k + 1, m - 1) \quad (k, m) \xrightarrow{(1-q)\omega} (k - 1, m + 1), \quad (10b)$$

where q controls the bias in the chipping process. Here, for simplicity, we consider fully left biased diffusion process, which as will be seen, is the relevant case for the traffic model under consideration. However, the results quoted below remain valid in the more general case where diffusion to the right is allowed as well. It has been shown [19–21] that if the chipping process is symmetric ($q = \frac{1}{2}$) there is a condensation transition at a critical occupancy ϕ_c , above which one site becomes macroscopically occupied. Furthermore, numerical simulations and mean-field studies show that the occupation probability p_k has the asymptotic form $p_k \sim z^k/k^\tau$ for large k , with $\tau = 5/2$. The parameter $z \leq 1$ is determined by the average particles occupancy and serves as the fugacity. The condensation transition is a result of the fact that $\tau > 2$, for which the distribution p_k cannot sustain high densities even at $z = 1$. This transition is analogous to the Bose-Einstein condensation. In contrast, if the chipping is asymmetric ($q \neq \frac{1}{2}$) there exists no phase transition at any occupancy [22]. In this case numerical studies indicate that the domain size distribution has the same form as above, but here $\tau = 2$. This distribution remains valid at any occupancy, with z approaching 1 as the average occupation is increased. Thus no condensation transition takes place.

Consider now the traffic model (9) in the deterministic limit $s = 1$. Within this limit it is straightforward to define a domain (or a jam) as a sequence of cars and isolated vacancies. These vacancies move deterministically to the left with velocity 1. The evolution of a domain can be described by two processes: (a) a chipping process, in which a car leaves the domain from its right end with rate u . Such a car leaves two vacant sites behind and is thus chipped off the domain; and (b) a diffusion process, in which a vacancy penetrates the domain from its right (with rate a) and advances deterministically to its left, thus shifting its center of mass one site to the right. In Fig. 2(a) a space-time configuration of the model at $a = 0.5, u = 0.1$ is given, focusing on a single domain. One readily observes the evolution of the domain through chipping and diffusion.

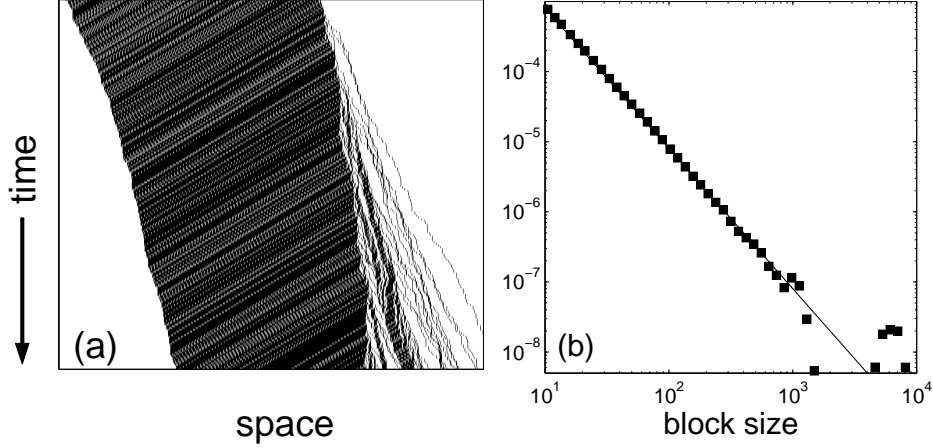


Fig. 2. (a) Space-time configurations of model (9), with $a = 0.5$ and $u = 0.1$. Cars are in black, vacancies in white. (b) Domain size distribution of the traffic model with $\rho = 0.5$, $a = 0.5$ and $u = 0.1$. The solid line has a slope -2 . Simulations were performed on a system of size $L = 20,000$ and averaged over 4×10^8 sweeps.

To test this picture we performed Monte-Carlo simulations of the traffic model and measured the domain size distribution p_k (Fig. 2(b)). We find that the asymptotic form of p_k is consistent with $k^{-\tau}$ with $\tau = 2$, as expected from the asymmetric CM. The existence of a macroscopic domain, suggested by the peak at large domains in Fig. 2(b), is thus merely a finite-size effect.

For $s < 1$, vacancies move inside the domain in a non-deterministic way. They can thus aggregate within a domain, introducing an additional process in the domain dynamics, namely breaking up of a domain into two or more domains of comparable size. This process clearly further reduce the probability of creating a macroscopic domain. It is thus concluded that the traffic model is not expected to exhibit a jamming transition at any rate s .

The fundamental diagram of this model as obtained by numerical simulations is given in Fig. 3. The diagram exhibits similar features as that of the simple model of the previous Section. Here, however, the role of cars and vacancies is reversed. In the deterministic limit $s = 1$ the absorbing state is found in the high density region, where vacancies move deterministically in a jam which spans the entire system. At low densities vacancies can aggregate, and the absorbing state is not reached. This is the region which corresponds to the asymmetric CM. In the intermediate region both states can coexist in the thermodynamic limit, as in the simple traffic model. Due to the reverse roles played by cars and vacancies, the fundamental diagram has a lambda-shape rather than the usual inverted-lambda. For $s < 1$ the current is a smooth function of the car density, and no transition is observed.

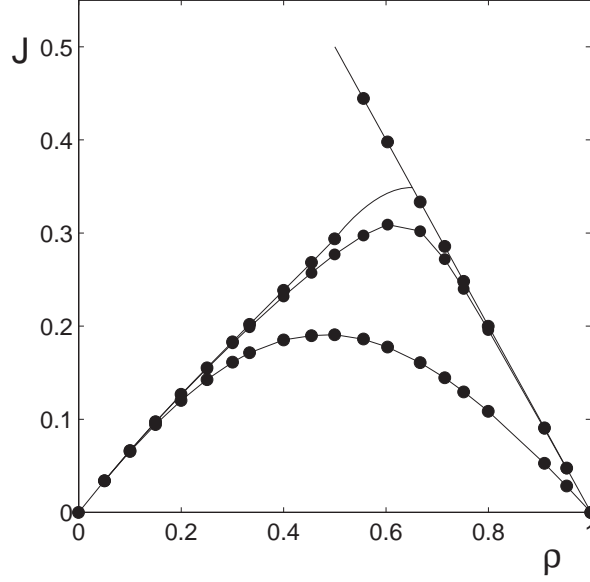


Fig. 3. The fundamental diagram of the extended model, for $a = 0.5, u = 0.1$ and $s = 0.6, 0.99$, and 1 from bottom to top. Simulations are carried out with $N = 100$ particles. Lines serve as a guide to the eye.

4 Other traffic Cellular Automata

It would be interesting to examine other traffic cellular automata within the framework of the chipping model. A particularly simple and instructive traffic model has recently been introduced in [24]. Using the same notation as above, the dynamics of this model is defined by the transition probabilities

$$\bullet \bullet \circ \circ \xrightarrow{\alpha} \bullet \circ \bullet \circ \quad \circ \bullet \circ \bullet \xrightarrow{\beta} \circ \circ \bullet \bullet \quad \bullet \bullet \circ \bullet \xrightarrow{\gamma} \bullet \circ \bullet \bullet \quad \circ \bullet \circ \circ \xrightarrow{\delta} \circ \circ \bullet \circ . \quad (11)$$

The characteristics of traffic flow of this model vary considerably with the four dynamical rates of the model. In different limits it exhibits wide moving jams, synchronized flow, convoys and other features which can be identified in real-life traffic. In the following we briefly comment on some regions of this phase space.

The case $\alpha = \gamma = r, \beta = \delta = q$ corresponds to the simplified model introduced in Section 2. Moreover, the case $\beta = \delta = 1$ is qualitatively similar to the case $q = 1$ even when $\alpha \neq \gamma$ [24]. Thus, in this region of the parameter space the coarse-grained dynamics of the model is characterized by chipping with no diffusion.

Another interesting case is $\delta = 1$, which is the cruise-control limit of the model. It has recently been argued that in this case the coarse-grained dynamics is described by the CM [16]. This correspondence is made by identifying free-flow domains in typical configurations of the model, whose dynamics is

characterized by both chipping and diffusion processes. It has also been verified numerically that the gap-size distribution for large gaps scales as k^{-2} , as expected from the CM. With $\delta < 1$ domains can break in their bulk. One thus expects a stronger suppression of large domains.

Note that if instead of $\delta = 1$ one considers $\gamma = 1$, the role played by cars and vacancies is interchanged. In this case domains correspond to jams, within which deterministic vacancies are embedded. In this case the distribution of these domains (or jams) behaves as k^{-2} , and thus no macroscopic jam is expected.

A very interesting region in the parameter space of this model is the case $\gamma = \delta = 1$, corresponding to the cruise-control limit which is symmetric under car-vacancy exchange (SCC). Here the model exhibits a low-density absorbing state at $\rho < 1/3$ and a high-density absorbing state at $\rho > 2/3$. It has been suggested [24] that the system exhibits a macroscopic jam at intermediate densities ($1/3 < \rho < 2/3$) for a particular region of the α, β -plane, indicating a jamming phase transition at some density. Due to the fact that both types of domains, the high-density and the free-flow ones, are deterministic, it is possible that this case lies beyond the class of traffic models described by the CM picture. It would be of interest to probe this region, as well as other regions of the parameter space, and study them in the context of the CM picture.

5 Summary

The recently suggested correspondence between traffic dynamics and the chipping model is reviewed and closely examined in two cases. The first exhibits chipping but no diffusion. This model is mapped onto a zero range process (ZRP), which is exactly soluble. The second is a more general model, whose coarse-grained domain dynamics exhibit both chipping and diffusion. In both cases it is shown that when all dynamical rates are non-deterministic the models do not exhibit a jamming phase transition, but rather a smooth crossover into the jammed state.

The approach reviewed in this paper could provide a useful tool for analyzing the behavior of a broader class of traffic models. Starting from a particular traffic model one should first identify the *domains* which characterize the flow. A domain can either be a low density segment, termed *a gap* or *a hole* in some studies ; a high density segment, termed *a jam* ; or a segment of some other characteristics, defined ad-hoc. A domain of size k is then associated with a site of the CM occupied by k particles. One then proceeds by examining the evolution of the domains, and identifying their dynamical processes. In many

traffic models with some deterministic rate these processes are the diffusion and the chipping processes of the asymmetric CM, implying that jamming transition does not take place. Usually when all dynamical rates are not deterministic other dynamical processes become possible, in which domains can break up into two or more pieces of comparable sizes. Such processes clearly disfavor the formation of macroscopic domains. We thus conclude that jamming transitions are not expected in these models.

A particularly instructive model is the one introduced in [24]. The correspondence to the chipping model has been demonstrated in some regions of its parameter space. It would be interesting to examine more closely other regions of the parameter space of this model, as well as other traffic models, in more details.

A different mechanism for condensation transition has been suggested to take place in some one-dimensional driven systems [2, 28, 29]. Here the dynamical process involves only chipping without diffusion. However, in these ZRP type processes, the chipping rates must decay slowly enough with the occupation of the departure site for condensation to take place. Condensation is obtained even when all dynamical processes are noisy. It would be of interest to examine the relevance of this mechanism to traffic models. This mechanism could lead to a jamming transition, which is absent in the asymmetric chipping model.

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